South Carolina College- and Career-Ready Standards for Mathematics

7th Grade Support Document

South Carolina Department of Education
Office of Standards and Learning
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South Carolina College- and Career-Ready Standards for Mathematics
Grade 7 Overview

The Table of Contents below arranges the *South Carolina College- and Career-Ready Standards for Mathematics* for middle school into *Course Coversheets* and *Units*.

- Each middle school *Course Coversheet* organizes the middle school course standards into possible instructional units and provides links to specific middle school course *Units*.
- Each middle school course *Unit* contains:
  - Clarifying notes related to the standards within the unit
  - New academic vocabulary in the unit
  - Prior and subsequent knowledge related to the unit
  - Description of the relationship between the standards in the unit
  - Potential instructional strategies and lessons
  - Resources for the unit
  - Sample formative assessment tasks and questions

South Carolina College- and Career-Ready Standards for Mathematics
Grade 7 Overview

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South Carolina College- and Career-Ready Standards for Mathematics
Middle School Support Document
Grade 7 Coversheet

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Standards	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**	**Standards**
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Mathematical Process Standards: The South Carolina College- and Career-Ready (SCCCR) Mathematical Process Standards demonstrate the ways in which students develop conceptual understanding of mathematical content and apply mathematical skills. As a result, the SCCCR Mathematical Process Standards should be integrated within the SCCCR Content Standards for Mathematics for each grade level and course. Since the process standards drive the pedagogical component of teaching and serve as the means by which students should demonstrate understanding of the content standards, the process standards must be incorporated as an integral part of overall student expectations when assessing content understanding.

a. Make sense of problems and persevere in solving them. a. Relate a problem to prior knowledge. b. Recognize there may be multiple entry points to a problem and more than one path to a solution. c. Analyze what is given, what is not given, what is being asked, and what strategies are needed, and make an initial attempt to solve a problem. d. Evaluate the success of an approach to solve a problem and refine it if necessary.	**5. Use a variety of mathematical tools effectively and strategically.** a. Select and use appropriate tools when solving a mathematical problem. b. Use technological tools and other external mathematical resources to explore and deepen understanding of concepts.
2. Reason both contextually and abstractly. a. Make sense of quantities and their relationships in mathematical and real-world situations. b. Describe a given situation using multiple mathematical representations. c. Translate among multiple mathematical representations and compare the meanings each representation conveys about the situation. d. Connect the meaning of mathematical operations to the context of a given situation.	**6. Communicate mathematically and approach mathematical situations with precision.** a. Express numerical answers with the degree of precision appropriate for the context of a situation. b. Represent numbers in an appropriate form according to the context of the situation. c. Use appropriate and precise mathematical language. d. Use appropriate units, scales, and labels.
3. Use critical thinking skills to justify mathematical reasoning and critique the reasoning of others. a. Construct and justify a solution to a problem. b. Compare and discuss the validity of various reasoning strategies. c. Make conjectures and explore their validity. d. Reflect on and provide thoughtful responses to the reasoning of others.	**7. Identify and utilize structure and patterns.** a. Recognize complex mathematical objects as being composed of more than one simple object. b. Recognize mathematical repetition in order to make generalizations. c. Look for structures to interpret meaning and develop solution strategies.
4. Connect mathematical ideas and real-world situations through modeling. a. Identify relevant quantities and develop a model to describe their relationships. b. Interpret mathematical models in the context of the situation. c. Make assumptions and estimates to simplify complicated situations. d. Evaluate the reasonableness of a model and refine if necessary.	

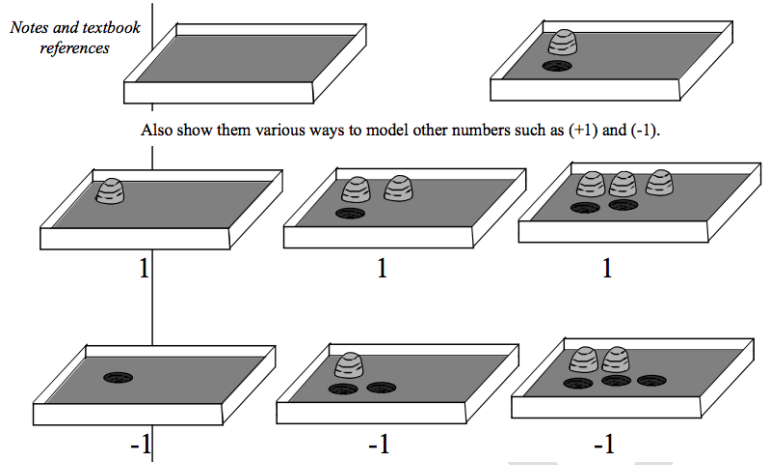
Unit Title
Rational Numbers
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.NS.1 Extend prior knowledge of operations with positive rational numbers to add and to subtract all rational numbers and represent the sum or difference on a number line. a. Understand that the additive inverse of a number is its opposite and their sum is equal to zero. b. Understand that the sum of two rational numbers ($p + q$) represents a distance from p on the number line equal to $\|q\|$ where the direction is indicated by the sign of q. c. Translate between the subtraction of rational numbers and addition using the additive inverse, $p - q = p + (-q)$. d. Demonstrate that the distance between two rational numbers on the number line is the absolute value of their difference. e. Apply mathematical properties (e.g., commutative, associative, distributive, or the properties of identity and inverse elements) to add and subtract rational numbers. o Understand, apply, and explain the additive inverse o Model addition and subtraction of rational numbers, including integers, decimals, and fractions using visual models **7.NS.2** Extend prior knowledge of operations with positive rational numbers to multiply and to divide all rational numbers. a. Understand that the multiplicative inverse of a number is its reciprocal and their product is equal to one. b. Understand sign rules for multiplying rational numbers. c. Understand sign rules for dividing rational numbers and that a quotient of integers (with a non-zero divisor) is a rational number. d. Apply mathematical properties (e.g., commutative, associative, distributive, or the properties of identity and inverse elements) to multiply and divide rational numbers. e. Understand that some rational numbers can be written as integers and all rational numbers can be written as fractions or decimal numbers that terminate or repeat. o Understand, apply, and explain the multiplicative inverse o Model multiplication and division of rational numbers, including integers, decimals, and fractions using visual models o Explain why division by zero is undefined **7.NS.3** Apply the concepts of all four operations with rational numbers to solve real-world and mathematical problems. o Model and solve real-world problems using numbers and operations o Explain the solution to a real-world problem in the context of the problem o Interpret the reasonableness of solutions in the context of the problem

New Academic Vocabulary for This Unit
● Integer ● Positive ● Negative ● Additive inverse ● Zero pairs ● Multiplicative inverse ● Terminating decimal ● Repeating decimal

Prior Knowledge Required for this Unit
● Fluent with operations of positive rational numbers (4.NSBT.4, 5.NSBT.5, 5.NSF.1, 5.NSF.4, 6.NS.1, 6.NS.2, 6.NS.3) ● Understand and perform operations using absolute value (6.NS.7) ● Fluent understanding of mathematical properties (e.g., commutative associative, distributive, or the properties of identity and inverse elements)(3.ATO.5, 4.NSBT.6, 5.NSBT.6, 6.EEI.3, 6.EEI.4) ● Fluent with the Order of Operations with positive rational numbers (3.ATO.8, 4.ATO.3, 5.ATO.1, 6.EEI.1, 6.EEI.2)
Subsequent Knowledge Related to this Unit
This unit will lead to mastery of the Order of Operations involving the fraction bar as a grouping symbol with integers in Grade 7. These standards will also guide students when solving expressions, equations, and inequalities with rational coefficients. The introduction of terminating and repeating decimals will extend to an understanding of 7.NS.5 involving the translation among multiple representations of rational numbers excluding a repeating decimal to a fraction. In Grade 8, students will continue to solve linear equations and inequalities with rational numbers; mastery of these skills will lay a strong foundation for success in high school math courses. In Grade 8, students will extend their knowledge to multiple representations of rational and irrational numbers.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop computational skills necessary for work with rational numbers.

Potential Instructional Strategies/Lessons

- Visual models
  - Two color counters
  - Addition and Subtraction
    - Heaps and Holes: Explain to your students that +1 is like a pile (or heap) of sand on a level beach. A hole of equivalent size dug into the beach represents -1. This model explains positives as a surplus and negatives as a deficit. (See page 9)



Source: NC Grade 7 Classroom Strategies

  - Multiplication

Source: Brain Camp

  - Number line (horizontal and vertical)

- Tracking Text: This strategy is used to process a word problem. The process entails underlining essential information needed to solve the given problem, including the labels as well as circling important math terms and renaming them with known vocabulary to assist in understanding what process to follow when solving for the answer.

  Example: When the navy wants to test the depth of a submarine they complete the following steps. First they take the submarine to a depth of 150 meters negative below sea level followed by moving up towards positive the surface 100 meters. The final step is to go deeper negative another 350 meters from where they were. How deep will the submarine be after this series of movements under water?

  -150m + 100m + (– 350m)

Resources
6–8 Progression on The Number System; High School, Number - http://commoncoretools.me/wp-content/uploads/2013/07/ccssm_progression_NS+Number_2013-07-09.pdf *Elementary and Middle School Mathematics: Teaching Developmentally*, Eighth Edition (older editions can be used as well) By John A. Van de Walle, Karen S. Karp, and Jennifer M. Bay-Williams ISBN-10: 0132612267; ISBN-13: 9780132612265 7.NS.1 - Through this unit, students will be able to analyze the addition and subtraction of integers by discussing the rise and fall of a hot air balloon. http://www.supermathunits.com/files/hot_air_ballon_unit.pdf 7.NS.2 - This document provides station-based activities for the classroom to assist students with the multiplication and division of rational numbers. http://moodle.wbrschools.net/pluginfile.php/3830/mod_resource/content/1/Set%202%20Multiplying%20and%20Dividing%20Rational%20Numbers.pdf 7.NS.1- 7.NS.3 - This Jeopardy game provides students with a computational review of operations with rational numbers. http://www.math-play.com/7th-Grade-Numbers-and-Operations-Jeopardy/7th-Grade-Numbers-and-Operations-Jeopardy.html
Sample Formative Assessment Tasks/Questions
Performance Task 7.NS.1: Jennifer decided to open a bank account with $150. On Monday she made a deposit of $25. The following week she made a withdrawal of $150. The next month there was an emergency and she needed to withdraw $200. After this last withdrawal does Jennifer have a positive or negative balance? Explain. Answer: Jennifer would have a negative balance in her account after the last withdrawal. You find this out by adding $25 to $150 to get a balance of $175. You then subtract the withdrawal of $150 from $175 to get a balance of $25. Finally you would subtract $200 from $25 to get a negative balance of -$175.

7.NS.2: A water well drilling rig has dug 60 feet below the surface of the water after one full day (24 hrs) of continuous use. A. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours? Answer: -37.5 feet B. If the rig has been running constantly and is currently at a height of -143.6 feet, for how long has the rig been running? Answer: approximately 2.93 hours
7.NS.2: The cheerleaders at a local middle school want new uniforms for the yearly cheerleading competition in South Carolina. A parent donated 20 ¾ yards of material. If each uniform takes ⅞ yard to make, how many uniforms can they make from the donated material? Explain your reasoning. Answer: 23 uniforms - If the total yardage is 20 ¾ yards and each uniform requires ⅞ yard, you must divide 20 ¾ by ⅞. The answer is 23 5/7 but because we are talking about uniforms, only 23 uniforms can be made from the material.
7.NS.3: Justin has two dogs, Finley and Noni. Each day Finley eats ⅓ of a can of dog food, and Noni eats ⅘ of a can. Dog food is only sold in 4 can packages, and each package costs $3. How much will Justin pay for a 30 day supply of dog food? Justify your reasoning. Answer: $27 - First, students must determine how many cans of dog food Justin will need for 30 days. By multiplying ⅓ of a can by 30 days, we see Finley requires 10 cans of dog food. By doing the same with ⅘ of a can and 30 days, we see Noni needs 24 cans of dog food. This means Justin will need 34 total cans of dog food for 30 days. If he can only buy cans of dog food in packs of 4, students must divide 4 into 34 to determine how many packages Justin must buy. The resulting quotient is 8 ½; however, individuals cannot buy partial packages of dog food. Therefore, Justin must buy 9 packages of dog food at $3 per package for a total price of $27.
7.NS.3: Iced Tea Stand - During this task, students will solve real-world and mathematical problems involving the four operations with rational numbers. They will also translate those operations into words/stories. Source: Howard County Public Schools
7.NS.3: A 20-foot piece of rope will be cut into as many 1 ½ foot sections as possible. How much rope will be left over? Answer: 6 inches; ½ foot

Unit Title
Expressions and Equations
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.EEI.1 Apply mathematical properties (e.g., commutative, associative, distributive) to simplify and to factor linear algebraic expressions with rational coefficients. ○ Use properties to simplify and factor algebraic expressions, including those with rational coefficients ○ Use the greatest common factor to factor algebraic expressions ○ Foundational formulas (e.g., perimeter, area) should be used to extend knowledge of algebraic expressions **7.EEI.2** Recognize that algebraic expressions may have a variety of equivalent forms and determine an appropriate form for a given real-world situation. ○ Describe real-world situations and determine the appropriate form of an algebraic expression (e.g., 2(x+4) or 2x+8) **7.EEI.3** Extend previous understanding of Order of Operations to solve multi-step real-world and mathematical problems involving rational numbers. Include fraction bars as a grouping symbol. ○ Understand that the fraction bar implies parentheses for the numerator ○ Order of Operations should include multiplication and division of rational numbers, including integers, decimals, and fractions **7.EEI.4** Apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations. a. Write and fluently solve linear equations of the form $ax + b = c$ and $a(x + b) = c$ where a, b, and c are rational numbers. b. Write and solve multi-step linear equations that include the use of the distributive property and combining like terms. Exclude equations that contain variables on both sides. c. Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning. d. Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities. ○ Equations must remain balanced ○ Use Order of Operations in the reverse order to isolate the variable **7.EEI.5** Understand and apply the laws of exponents (i.e., product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property) to simplify numerical expressions that include whole-number exponents. ○ Understand that a number raised to a power of zero results in a value of 1, not 0 ○ Derive the laws of exponents (i.e., product rule, quotient rule, power to a power, product to a power, quotient to a power, zero power property) before applying the laws in numerical expressions

7.NS.4 Understand and apply the concepts of comparing and ordering to rational numbers.

a. Interpret statements using less than (<), greater than (>), less than or equal to (≤), greater than or equal to (≥), and equal to (=) as relative locations on the number line.
b. Use concepts of equality and inequality to write and explain real-world and mathematical situations.
   - Understand that a closed circle represents the rational number and an open circle will not represent the rational number when graphing an inequality
   - Focus on real-world situations including vocabulary for less than or equal to (≤) and greater than or equal to (≥)

New Academic Vocabulary for This Unit
• Product Rule • Quotient Rule • Power to a Power • Product to a Power • Quotient to a Power • Zero Power Property • Greater than or Equal to • Less than or Equal to

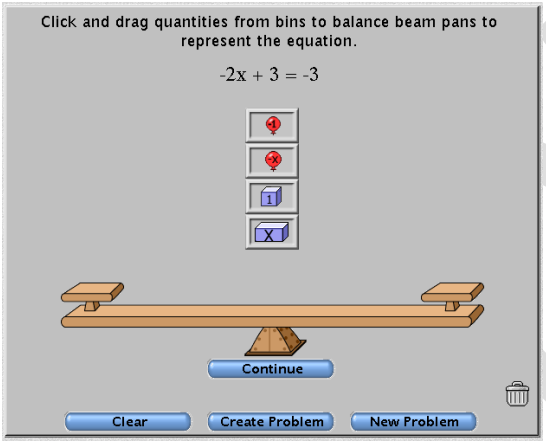
Prior Knowledge Required for this Unit
• Evaluate numerical expressions involving grouping symbols (5.ATO.1) • Write and evaluate numerical and algebraic expressions with whole number exponents and positive rational numbers (6.EEI.1, 6.EEI.2) • Apply mathematical properties to generate and justify equivalent expressions (6.EEI.3, 6.EEI.4) • Calculate the greatest common factor of two numbers less than or equal to 100 (6.NS.4)
Subsequent Knowledge Related to this Unit
In Grade 8, students will extend knowledge of the Laws of Exponents to include numerical expressions with integer exponents. In high school courses, students will apply the Laws of Exponents with rational exponents and algebraic expressions. Additionally in Grade 8, students will solve multi-step equations with variables on both sides of the equal sign. Grade 7 students will use their complete understanding of the Order of Operations and their properties to lay a foundation for functions in Grade 8.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop the computational skills needed for work within the real number system including solving multi-step linear equations and inequalities and simplifying expressions which include the use of whole-number exponents.

Potential Instructional Strategies/Lessons

- Algebra Tiles

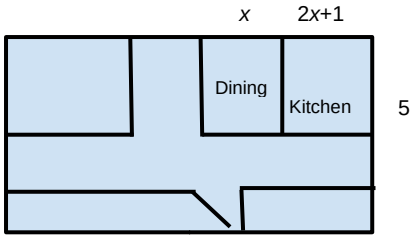
Source: National Library of Virtual Manipulatives

- Equations Balance



Source: National Library of Virtual Manipulatives

- Hands-on Equations

Resources
Progression document for Grade 6- Grade 8 expressions and equations: https://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf Solving multi-step equations: http://illuminations.nctm.org/Activity.aspx?id=3482 : this website allows the students to solve multi step equations using algebra tiles. 7.EEI.2 - Balance Expressions - http://illuminations.nctm.org/Lesson.aspx?id=2747 7.EEI.4 - This website provides a Jeopardy review game for students to practice the skills involved in solving multi-step equations and simplifying expressions. https://jeopardylabs.com/play/solving-multi-step-equations1 7.EEI.5 - This website demonstrates how to derive the laws of exponents. Use this website as a guide for lesson planning. http://www.mathsisfun.com/algebra/exponent-laws.html
Sample Formative Assessment Tasks/Questions
7.EEI.1: The Martin family is remodeling their home. They are removing the wall between the kitchen and the dining room. Use the model below to determine the area of the new room.    Answer: 15x + 5 square units - Since the wall is being removed between the dining room and kitchen, students should add the lengths “x” and “2x+1” to determine the length of the newly formed room: 3x + 1 units. To determine the area, students must multiply the length by the width (or base by the height), so they should use the given width of 5 units, which is consistent for both the dining room and kitchen. By multiplying 5 by 3x + 1, it can be determined that the area of the newly formed room is 15x + 5 square units.

7.EEI.1: The Quilt of a Math Teacher - This task will assess students' knowledge of formulating an algebraic expression based on a real-life situation

Source: New York City Department of Education

7.EEI.2: This task has students become the business owner of a store and business is a little slow. Since a sale is the best way to attract customers, they have to determine which sale is best and justify their reasoning.

Source: Howard County Public Schools

7.EEI.3: In this task, students will use formulas to compute the number of points each player and team totals in an NFL Weekend for a fantasy football league.

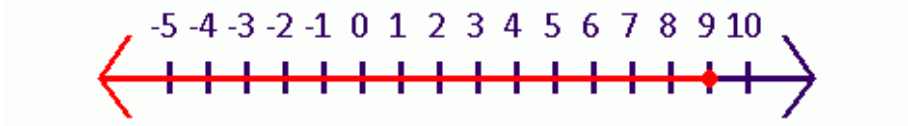
Source: Yummy Math

7.EEI.4: Thompsons Water Retreats rents boats for weekend trips. Each boat can carry 1742 pounds of people and luggage. The average weight of a person is 153 pounds, and each person brings a 40 pound suitcase.

Write an inequality to describe the restrictions on the number of people possible in a boat.

a. What is the maximum number of people that can rent a boat together?
b. Several families want to rent boats for July 4th. The first family has 4 people, the second has 11 people, and a third has 9. Which groups, if any, can rent a boat?
c. Construct a graph to represent the solution set.

Answers:

a. $x(153 + 40) \leq 1742$ or $193x \leq 1742$
b. 4 people - yes; 11 people - no; 9 people -yes - By solving for x, one gets the answer $x \leq 9.03$; this means total family members on the boat must be less than this number.
   9 people - If $x \leq 9.03$, then a maximum of 9 people can be on the boat together.
c. 

7.EEI.4: This task challenges a student to use algebra to represent, analyze, and generalize a variety of functions including linear relationships.

Source: Mathematics Assessment Resource Service

7.EEI.5: Solve the following expression: $\frac{3^0 \cdot (4^2)2}{2^2 \cdot 2^2}$

Answer: the solution to this question is 2. The first step would be to square 4 and get the value 16 and to rewrite the denominator as 2^4. Next would be to evaluate 3^0 as 1 and evaluate 2^4 as 16. Finally in the numerator multiply 16 and 2 to get 32 then divide 32 by 16 and get the final answer 2.

7.NS.4: Camille, the band director for the Middletown High School marching band, is buying instruments to expand the brass section. A tuba costs $960. The total expenditure must be below $2,700.

Write an inequality that describes this situation. Use the given numbers and the following variable.
x = the number of tubas purchased

Answer: $960x < 2700$

7.NS.4: Ning is printing out copies of a presentation. It takes 5 minutes to print a color copy. He wants to spend a maximum of 50 minutes on printing.

Write an inequality that describes this situation. Use the given numbers and the following variables.
x = the number of color copies

Answer: $5x \leq 50$

Unit Title
Ratios and Proportions
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.RP.1 Compute unit rates, including those involving complex fractions, with like or different units. ○ Realize a complex fraction indicates a fraction within a fraction (e.g., $\frac{\frac{1}{2}}{\frac{1}{4}}$) **7.RP.2** Identify and model proportional relationships given multiple representations, including tables, graphs, equations, diagrams, verbal descriptions, and real-world situations. a. Determine when two quantities are in a proportional relationship. b. Recognize or compute the constant of proportionality. c. Understand that the constant of proportionality is the unit rate. d. Use equations to model proportional relationships. e. Investigate the graph of a proportional relationship and explain the meaning of specific points (e.g., origin, unit rate) in the context of the situation. ○ Distinguish between linear and proportional relationships. ○ Understand that proportional relationships are linear and include the origin in the solution set. **7.RP.3** Solve real-world and mathematical problems involving ratios and percentages using proportional reasoning (e.g., multi-step dimensional analysis, percent increase/decrease, tax). ○ Include sales tax, tip, markup/discount, simple interest, depreciation, commission, percent error ○ Solve for all values in a simple interest problem ○ Calculate the percent increase/decrease by $\frac{\lvert New-Original\rvert}{Original} = \frac{Part\ of\ Percent}{100}$ **7.NS.5** Extend prior knowledge to translate among multiple representations of rational numbers (fractions, decimal numbers, percentages). Exclude the conversion of repeating decimal numbers to fractions. ○ Recognize that $\frac{1}{3} = 0.\overline{3}\ and\ \frac{2}{3} = 0.\overline{66}$.

7.EEI.4 Apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.

a. Write and fluently solve linear equations of the form $ax + b = c$ and $a(x + b) = c$ where $a, b, and\ c$ are rational numbers.

b. Write and solve multi-step linear equations that include the use of the distributive property and combining like terms. Exclude equations that contain variables on both sides.

c. Write and solve two-step linear inequalities. Graph the solution set on a number line and interpret its meaning.

d. Identify and justify the steps for solving multi-step linear equations and two-step linear inequalities.

- Include equations and inequalities with rational number coefficients, constants, and solutions.
- Justify steps and solutions using multiple means (e.g., substitution, properties, definitions)

7.GM.1 Determine the scale factor and translate between scale models and actual measurements (e.g., lengths, area) of real-world objects and geometric figures using proportional reasoning.

- Include examples for scaling up and scaling down (e.g., a scale model of an eye would be scaling up, a map would be an example of scaling down).

New Academic Vocabulary for This Unit

- Proportion
- Proportional relationships
- Complex fraction
- Constant of proportionality
- Like Terms
- Linear inequalities
- Scale factor
- Scale model

Prior Knowledge Required for this Unit
• Ratios, rates, and unit rates (6.RP.1, 6.RP.2, 6.RP.3, 6.EEI.9) • Expressions, equations, and inequalities (6.EEI.2b, 6.EEI.3, 6.EEI.4, 6.EEI.5, 6.EEI.7, 6.EEI.8, 6.EEI.9) • Translations among rational numbers (6.NS.9) • Scale factor of geometric shapes (6.GM.1)
Subsequent Knowledge Related to this Unit
In Grade 6, students determine the basic meaning ratios and unit rates, leading to complex ratios and rates in Grade 7. Subsequently, in Grade 8, students will apply unit rates to functions leading to an understanding of constant rate of change and slope. In high school mathematics courses, students will extend the knowledge of constant rate of change to include rate of change and average rate of change. In Grade 6, students solved problems involving one-step dimensional analysis. In Grade 7 and beyond, multi-step dimensional analysis is taught, used, and applied beyond mathematics courses (e.g., chemistry, physics). In Grade 6, students begin to translate among representations of rational numbers with limited denominators (2, 3, 4, 5, 8, 10, and 100), which is extended to all denominators in Grade 7. In Grade 8, students will extend their knowledge of translations among rational numbers to include the conversion of repeating decimal numbers to fractions. In Grade 6, students created and solved one-step equations and inequalities. In Grade 7, students create and solve multi-step linear equations and inequalities, utilizing the distributive property and combining like terms. Students extend knowledge of determining a solution set in Grade 6 to graph and interpret solutions to multi-step linear inequalities in Grade 7. In Grade 8, students solve linear equations and inequalities to include variables on both sides and systems of equations. In high school courses, students will create, solve, and graph linear and nonlinear equations and inequalities in two variables. Additionally, students will apply knowledge and understanding of solving equations to solve literal equations for an indicated value. In Grade 7, scale factor and scale model are introduced using proportional reasoning which leads to, in Grade 8, an understanding of the relationship between dilations and attributes of geometric figures. In high school, dilations will be extended to include negative scale factors, which represent reflections of enlargements and reductions.
Relationship Among Standards in this Unit
Standards in this unit are all necessary to develop an understanding of proportional relationships and model those relationships using multiple representations.

Potential Instructional Strategies/Lessons

- **Real-world Problems**

Example 1:
If $\frac{1}{2}$gallon of paint covers $\frac{2}{5}$of a wall, then how much paint is needed for the entire wall?

Answer
½ gallon / ⅖ of a wall
1 ¼ gallons per wall.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Example 2:
The table below gives the price for different numbers of books. Do the numbers in the table represent a proportional relationship? Justify your reasoning.

Number of Books (x)	Price (y)
1	3
3	9
4	12
7	18

Answer
Students can examine the numbers to determine that the price is the number of books multiplied by 3,except for 7 books. The row with seven books for $18 is not proportional to the other amounts in the table therefore, the table does not represent a proportional relationship. Students graph relationships to determine if two quantities are in a proportional relationship and to interpret the ordered pairs. If the amounts from the

table above are graphed (number of books, price), the pairs (1, 3), (3, 9), and (4, 12) will form a straight line through the origin (0 books,0 dollars), indicating that these pairs are in a proportional relationship. The ordered pair (4, 12) means that 4 books cost $12. However, the ordered pair (7, 18) would not be on the line, indicating that it is not proportional to the other pairs.

The ordered pair (1, 3) indicates that 1 book is $3, which is the unit rate. The y-coordinate when x= 1 will be the unit rate. The constant of proportionality is the unit rate. Students identify this amount from tables (see example above), graphs, equations and verbal descriptions of proportional relationships.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Example 3:
Part A:
The price of bananas at Trader Joe's can be determined by the equation: P= $0.35n, where P is the price and n is the number of pounds of bananas. What is the constant of proportionality (unit rate)?

Part B:
Pete shopped for bananas at Publix on three separate occasions. The table below illustrates the prices Pete paid for the bananas. Determine the cost for 1 pound of bananas.

Number of Pounds of Bananas (x)	Total Price (y)
3 lb	$1.20
4 lb	$1.60
6 lb	$2.40

Answer
Part A: The constant of proportionality is the coefficient of x (or the independent variable). The constant of proportionality is 0.35.
Part B: The price per pound of bananas is $0.40. The unit rate represents the constant of proportionality when the rate is represented by a denominator of 1.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Example 4:
Sally has a recipe that needs ¾ tablespoon of butter for every 2 cups of milk. If Sally decreases the amount of milk to 1 cup of milk, how many tablespoons of butter are needed?

Using these numbers to find the unit rate may not be the most efficient method. Students can set up the following proportion to show the relationship between butter and milk.

$$\frac{\frac{3}{4}}{2} = \frac{x}{1}$$

Answer
One possible solution is to recognize that $2 \div 2 = 1$ (or $2 \times \frac{1}{2}$) and that $\frac{3}{4} \div 2 = x$ (or $\frac{3}{4} \times \frac{1}{2}$). The amount of butter needed would be $\frac{3}{8}$ tablespoons.

A second way to solve this proportion is to use cross-multiplication $\frac{3}{4} \cdot 1 = 2x$. Solving for *x* would give $\frac{3}{8}$ tablespoons of butter.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Example 5:
Kohl's is having a 30% off sale on their sweaters. If the original price was $37.50, what is the sale price of the sweater before sales tax?

Answer
The discount is 30% of $37.50, or $11.25. The sale price of the sweater is the original price minus the discount, $37.50 - $11.25, or $26.25.
Alternately, complements can be used to calculate the sale price. If the discount is 30% off, the sale price is 70% of the original cost, 0.70 x $37.50.
Sale Price = 0.70 × Original Price.

Resources
7.GM.1 **-** This activity allows students to practice working with scale factors. http://illuminations.nctm.org/Activity.aspx?id=4207 7.EE1.4 - This game allows students an opportunity to graph simple linear inequalities. http://ltcconline.net/greenl/java/BasicAlgebra/inequalityGame/inequalities.html 7.EE1.4 - This game helps students understand the meaning of inequalities. http://www.xpmath.com/forums/arcade.php?do=play&gameid=87 7.RP.2 - This game allows an opportunity to solve proportions. http://www.xpmath.com/forums/arcade.php?do=play&gameid=97 7.EE1.4 - This game allows students to practice recognizing like terms. http://www.xpmath.com/forums/arcade.php?do=play&gameid=92 7.RP.2 - This activity checks for an understanding of constant of proportionality. http://www.mathopolis.com/questions/q.php?id=8943&site=1&ref=/algebra/directly-inversely-proportional.html&qs=8943_8945_8947_8948_8951_8952_7005_8064_8065_8067_8072 7.NS.5 - This game allows students to practice equivalences between decimals, fractions, and percents. http://www.mathplayground.com/Decention/Decention.html 7.RP.1 - This Jeopardy games gives students an opportunity to practice finding unit rates and make comparisons for the best price. http://www.quia.com/cb/195515.html

Sample Formative Assessment Tasks/Questions

7.RP.1: Jamal is painting his room. He determines that a $\frac{1}{2}$ gallon container of paint will cover $\frac{1}{6}$ of a wall. How much paint is needed for an entire wall (assuming there are no doors or windows)?

Answer
If a wall is divided into sixths, there will be six sections. Jamal will need 6 half-gallon containers; this is equivalent to 3 gallons of paint.

7.RP.2: The Smart Start Healthy Breakfast Company is studying the amount of sugar in common breakfast foods. Their findings are shown in the table below.

Breakfast Item	Serving Size (g)	Sugar (g)
Strawberry Pop-Tart	52g	16.5g
Cinnamon Toast Crunch Cereal Bar	45g	14g
KrispyKreme Glazed Doughnut	49g	10g

a. How many grams of Strawberry Pop-Tart will give you 22 grams of sugar? (Round your answer to the nearest whole number.)
b. Which breakfast food gives the highest ratio of sugar?

Answer
a. 66 grams
b. Strawberry Pop-Tarts have the highest ratio of sugar. (0.3173)

7.RP.2, 7.RP.3:

a. The total cost spent on baseball bats by the Columbia Fireflies when shopping at DICK's Sporting Goods can be determined by the equation: $C = \$196n$, where C is the price and n is the number of baseball bats purchased. What is the constant of proportionality (unit rate)?
b. The Fireflies also shopped for baseball bats at Academy Sports on four separate occasions. The table below illustrates the prices paid for the baseball bats. Determine the cost for 1 baseball bat.

Number of Baseball Bats Purchased (x)	Total Cost of Baseball Bats (y)
5	890
9	1602
11	1958
14	2492

c. Write an equation to represent the total cost (C) of buying n bats at Academy Sports.
d. Which store do you expect the Fireflies to use when purchasing baseball bats for the team? Justify your answer.
e. What is the markup (percent increase) used by the more expensive store? (Round to the nearest percent.)
f. Can you think of any reasons the Fireflies may choose to buy baseball bats from the more expensive store?

Answer

a. The constant of proportionality (unit rate) is \$196.
b. One bat will cost \$178 at Academy Sports.
c. $C = 178n$
d. Academy Sports; The Columbia Fireflies will spend less money per bat.
e. 10%
f. Student answers will vary but may include store location, bat quality, or contractual agreements.

7.RP.3: At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs by giving all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify the solution.

Answer
The sales team members need to sell the original 48 plus an additional 30%. 100% of the TV sales is 48 while 30% is the number of TVs that need to be sold in addition to the 48. This leads to finding 130% of TVs sold to show an increase of 30% which is the same as multiplying 48 and 1.3 (130%) arriving at the answer of 63 TVs sold in May.

Alternate Solution: A second solution to this is to set up a proportion as a percent of increase question. $\frac{x}{48} = \frac{30}{100}$. After cross-multiplying to get the equation 100x=1440 then dividing by 100 to arrive at the answer 14.4. Since you cannot purchase a decimal portion of a TV you would need to round 14.4 to 15 TVs since we are looking for a full 30% increase in sales. Next would be to add the original 48 TVs and the 15 TVs increase in sales to a May.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

7.NS.5:
Complete the table with the appropriate fraction, decimal, or percent.

Fraction	Decimal	Percent
	0.83	
		32%
$\frac{22}{5}$		
		5.5%
	2.9	
		$\frac{3}{4}$%
$\frac{1}{6}$		

Answer

Fraction	Decimal	Percent
$\frac{83}{100}$	0.83	83%
$\frac{8}{25}$	0.32	32%
$\frac{22}{5}$	4.4	440%
$\frac{11}{200}$	0.055	5.5%
$2\frac{9}{10}$ or $\frac{29}{10}$	2.9	290%
$\frac{3}{400}$	0.0075	$\frac{3}{4}$%
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$%

7.EEI.4: Mr. Mann's class is going to the state fair. The trip costs $52. Included in that price is $11 for a concert ticket and the cost of 2 passes, one for rides and one for game booths. Each of the passes cost the same price.

a. Write an equation representing the cost of the trip.
b. Determine the price of one pass.

Answer

a. 52 = 2x + 11
b. Each pass costs $20.50.

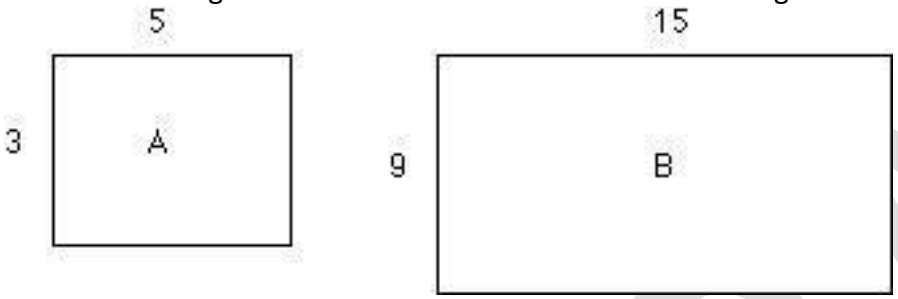
7.EEI.4: Florencia has at most $60 to spend on clothes. She wants to buy a pair of jeans for $22 and spend the rest on shirts. Each shirt costs $8.

a. Write an inequality for the number of t-shirts she can purchase.
b. Determine the number of shirts Florencia can purchase.

Answer

a. $60 > 22 + 8x$
b. $4.75 > x$, this means Florencia can buy 4 shirts.

7.GM.1: Rectangle B was reduced in size to become Rectangle A. What is the scale factor?

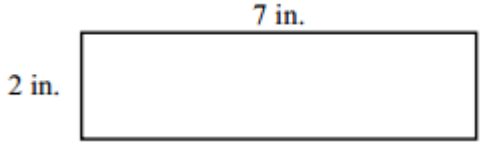


Answer

Set up a proportion $\frac{15}{9} = \frac{5}{3}$. Determine that you must divide 15 by 3 to get 5, therefore the scale factor is $\frac{1}{3}$.

Source: Watertown Public Schools

7.GM.1: If the rectangle below is enlarged using a scale factor of 1.5, what will be the perimeter and area of the new rectangle?



Perimeter _____

Area _____

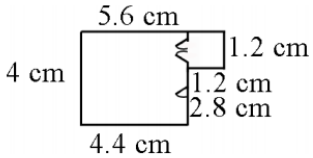
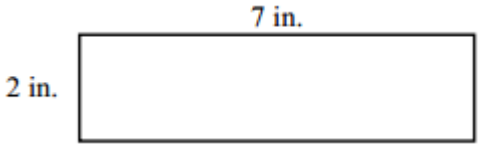
Answer
Perimeter = 27 *in.*
Area = 31.5 $in.^2$

Unit Title
Geometry
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.GM.1 Determine the scale factor and translate between scale models and actual measurements (e.g., lengths, area) of real-world objects and geometric figures using proportional reasoning. ○ Include examples for scaling up and scaling down (e.g., a scale model of an eye would be scaling up, a map would be an example of scaling down). ○ Make the distinction between a scale factor and scale. Scale factors are always expressed in the same units. (1cm to 50 cm) Scale is used when working with maps, drawings, or photographs and many times are expressed in two different units (1 inch = 10 miles). **7.GM.2** Construct triangles and special quadrilaterals using a variety of tools (e.g., freehand, ruler and protractor, technology). a. Construct triangles given all measurements of either angles or sides. b. Decide if the measurements determine a unique triangle, more than one triangle, or no triangle. c. Construct special quadrilaterals (i.e., kite, trapezoid, isosceles trapezoid, rhombus, parallelogram, rectangle) given specific parameters about angles or sides. ○ Two-dimensional figures that are constructed by freehand need not be drawn to scale. However, all measurements should be labeled. ○ Not all combinations of side measurements or angle measurements produce triangles. ○ The conditions for unique triangles are such that the dimensions will not satisfy another (unless they are congruent). ○ Some combinations of measurements will produce more than one triangle (For example, there are an infinite number of triangles that contain three 60°angles. Additionally, a triangle with side lengths 3 and 4 could have a third side that measures greater than 1 but less than 7.) ○ The lengths of any two sides of a triangle will have a sum greater than the third (Triangle Inequality Theorem). ○ Categorize triangles by the lengths of their sides (isosceles, equilateral, and scalene) as well as by the measure of their angles (right, obtuse, and acute). **7.GM.3** Describe two-dimensional cross-sections of three-dimensional figures, specifically right rectangular prisms and right rectangular pyramids. ○ Include horizontal, vertical, and diagonal cross-sections. ○ Predict the two-dimensional figures that result from slicing a 3-D figure such as a right-rectangular prism or pyramid. (perpendicular cuts and parallel cuts)

New Academic Vocabulary for This Unit
• Scale drawings • Scale factor • Cross-section • Freehand • Diagonal • Equiangular • Interior Angles

Prior Knowledge Required for this Unit
• Identify and describe two-dimensional shapes (K.G.2) • Identify triangles, quadrilaterals, hexagons, and cubes. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (2.G.1) • Understand that shapes in different categories (e.g., rhombus, rectangle, square, and other 4-sided shapes) may share attributes (e.g., 4-sided figures) and the shared attributes can define a larger category (e.g., quadrilateral). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories (3.G.1) • Identify a three-dimensional shape (i.e., right rectangular prism, right triangular prism, pyramid) based on a given two-dimensional net and explain the relationship between the shape and the net (3.G.4) • Recognize right triangles as a category, and identify right triangles. (4.G.3)
Subsequent Knowledge Related to this Unit
Constructions facilitate an understanding of geometry. Providing opportunities for students to construct angles, triangles, and quadrilaterals will allow students to discover the side and angle conditions that will form triangles. Students should be able to use this knowledge to determine if certain side lengths or angles can form a triangle and justify their conclusions with both sketches and reasoning. This leads directly into preparation for proofs in geometry. In high school mathematics courses, students will use their understanding of triangles' attributes to recognize similar and congruent triangles. Students will also draw conclusions about the perimeters, areas, and volumes of similar figures based on the scale factor that exists between them. Also in high school, students will extend their knowledge of cross sections to explain Cavalieri's Principle.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among the attributes of triangles, quadrilaterals, and three-dimensional figures (e.g. prisms and pyramids). They also help develop 3-D visualization skills.

Potential Instructional Strategies/Lessons
• Dynamic Geometric Software (e.g. Geometer's Sketchpad or GeoGebra) • Compass and Straightedge construction • Play Doh and floss for cross sections - Have students form three-dimensional figures with the Play Doh and "slice" the figures using dental floss to determine horizontal, vertical, and diagonal cross sections. •  • Geometric Solids and Nets
Resources
7.GM.1 - This video discusses scale factors (including those greater than and less than one). https://learnzillion.com/lesson_plans/7167-generate-a-scale-drawing-using-scale-factors-greater-than-and-less-than-one 7.GM.1 - This YummyMath activity has students use proportional reasoning to discuss and theorize about the size of various television screens. http://www.yummymath.com/2015/glowing-rectangles-2/ 7.GM.2 - This video covers the construction of polygons based on given characteristics. https://www.youtube.com/watch?v=YB4FhIcCHro 7.GM.3 - This site defines a cross-section and also has an interactive activity which allows you to slice and see the cross-sections of 3D figures. http://www.learner.org/courses/learningmath/geometry/session9/part_c/

Sample Formative Assessment Tasks/Questions
7.GM.1: Julie shows the scale drawing of her room below. If 2 cm. on the scale drawing equals 5 ft., what are the actual dimensions of Julie's room?    *Answer* 5.6 cm. → 14 ft. 1.2 cm. → 3 ft. 2.8 cm. → 7 ft. 4.4 cm. →11 ft. 4 cm. → 10 ft. Source: NC DPI 7th Grade Mathematics Unpacked Contents
7.GM.1: If the rectangle below is enlarged using a scale factor of 1.5, determine the perimeter and area of the new rectangle.    Perimeter _____ Area _____ *Answer* The original rectangle measures 2 in. x 7 in. The scaled drawing would increase to the dimensions of 3 in. x 10.5 in. Thus, the new perimeter will be 27 inches and the area will be 31.5 in.^{2}.

7.GM.1: Maurice has a 60:1 scale-drawing of the floor plan for his new restaurant. His floor plan shows that the dimensions for his kitchen are $1\frac{2}{3}$ inches by $3\frac{1}{4}$ inches. What is the actual area of his kitchen in square inches? Is this the most reasonable unit of measure to use, if no, what would be better?

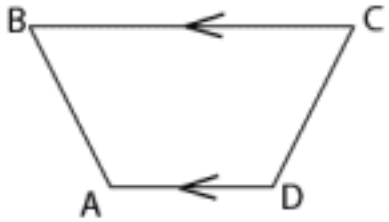
Answer
Using the ratio 60:1, we can determine that the actual kitchen is 100 inches by 195 inches. To determine the area in square inches we would multiply the two dimensions together and determine the area of the kitchen is 19,500 square inches. Square inches is not the best unit to use, square feet would have been a better option.

7.GM.2: Can a triangle have more than one obtuse angle? Explain your reasoning.

Answer
No, a triangle cannot have more than one obtuse angle because the interior angles of a triangle will have a sum of 180 degrees. If one angle is greater than 90 degrees, then the other two angles cannot have a sum to exceed 90 degrees. Therefore, there will only be one obtuse angle.

7.GM.2: Draw a quadrilateral with one set of parallel sides and no right angles.

Answer
A trapezoid is a quadrilateral that has exactly one pair of parallel sides. This trapezoid does not have any right angles.



7.GM.2: Will three sides of any length create a triangle? Explain how you know which will work.
Possibilities to examine are:

a. 13 cm, 5 cm, and 6 cm
b. 3 cm, 3 cm, and 3 cm
c. 2 cm, 7 cm, 6 cm

Answer
The sum of the lengths of any two sides of a triangle is greater than the length of the third side. If you take the three sides of a triangle and add them in pairs, the sum is greater than (not equal to) the third side. If this is not true, then it is not possible to construct a triangle with the given side lengths.

In the possibilities examined,

a. Based on the Triangle Inequality Theorem, the side lengths 13cm, 5cm, and 6cm, cannot create a triangle because the sum of 5cm and 6cm is 11cm which is not greater than the third side length of 13 cm.
b. Based on the Triangle Inequality Theorem, the side lengths of 3 cm, 3 cm, and 3 cm, can create a triangle. It would be considered an equilateral triangle due to all sides being equal.
c. Based on the Triangle Inequality Theorem, the side lengths of 2cm, 7cm, and 6 cm, can create a triangle. This triangle would be considered a scalene triangle, due to all unequal side lengths.

7.GM.3: Jordan is a master ninja and can slice straight through objects using nothing but his hands. Anxious to show off his skills, he approaches three boxes each in the shape of a cube. He slices through the first box horizontally, the second vertically, and the third diagonally. What two dimensional shape will each of the cross sections represent?

Answer
Horizontal: Square
Vertical: Square
Diagonal: Rectangle

Unit Title
Algebraic Geometry
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.GM.4 Investigate the concept of circles. a. Demonstrate an understanding of the proportional relationships between diameter, radius, and circumference of a circle. b. Understand that the constant of proportionality between the circumference and diameter is equivalent to π. c. Explore the relationship between circumference and area using a visual model. d. Use the formulas for circumference and area of circles appropriately to solve real-world and mathematical problems. ○ Understand that pi is derived from the ratio of the circumference of a circle and its diameter. ○ Recognize that πd and $2\pi r$ are equivalent expressions that can be used to find circumference. ○ Recognize 3.14 and $\frac{22}{7}$ as acceptable approximations for pi. ○ Recognize 2π as an exact value and 6.28 as an approximate answer. **7.GM.5** Write equations to solve problems involving the relationships between angles formed by two intersecting lines, including supplementary, complementary, vertical, and adjacent. ○ Recognize that two angles whose sum is $90°$ are complementary angles ○ Recognize that two angles whose sum is $180°$ are supplementary angles. ○ Recognize that vertical angles are congruent to one another. ○ Recognize that two pairs of vertical angles are formed when two lines intersect one another. ○ Recognize that two angles are defined as adjacent if they share a vertex and a side. ○ Recognize that two angles that are supplementary and adjacent form a linear pair.

7.GM.6 Apply the concepts of two- and three-dimensional figures to real-world and mathematical situations.
a. Understand that the concept of area is applied to two-dimensional figures such as triangles, quadrilaterals, and polygons.
b. Understand that the concepts of volume and surface area are applied to three dimensional figures such as cubes, right rectangular prisms, and right triangular prisms.
c. Decompose cubes, right rectangular prisms, and right triangular prisms into rectangles and triangles to derive the formulas for volume and surface area.
d. Use the formulas for area, volume, and surface area appropriately.

- Recognize that surface area covers a three-dimensional shape while volume fills the three-dimensional shape.
- Recognize that surface area is the sum of the areas of the bases and lateral surfaces of a three-dimensional figure.
- Recognize that the volume of cubes, right rectangular prisms,and right triangular prisms is equivalent to the area of the base multiplied by the height ($V = Bh, where\ B\ is\ the\ area\ of\ the\ base$).
- Recognize that surface area is labeled with square units and volume is labeled with cubic units.
- Recognize that a length is measured in linear units if finding an unknown length.

7.EEI.4 Apply the concepts of linear equations and inequalities in one variable to real-world and mathematical situations.
b. Write and solve multi-step linear equations that include the use of the distributive property and combining like terms. Exclude equations that contain variables on both sides.

- Write and use equations to solve problems involving circumference, area, surface area, volume, and angle relationships.

New Academic Vocabulary for This Unit

- Diameter
- Radius
- Circumference
- Supplementary Angles
- Complementary Angles
- Vertical Angles
- Adjacent Angles
- Linear Pair
- pi (π)

Prior Knowledge Required for this Unit
• Solve 2-step equations (7.EEI.4) • Understand the constant of proportionality (7.RP.2) • Understand how to use formulas to find area, surface area, volume (4.MDA.3, 5.MDA.3, 6.GM.2, 6.GM.4)
Subsequent Knowledge Related to this Unit
In high school mathematics classes, students will use their knowledge of angle measures to prove theorems about two- and three-dimensional figures and extend their knowledge of circles to determine the lengths of arcs and the area of sectors within a circle. Students will also use a three-dimensional figure's apothem and perimeter to determine its area. Also in high school, students will apply Cavalieri's Principle to prove the volumes of figures possessing the same height and cross-sectional areas are equivalent.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among the concepts of two- and three- dimensional figures.

Potential Instructional Strategies/Lessons

• **Real-world Connections** - use circular items and have students measure the circumference and the diameter to help students visualize the ratio between circumference and diameter which is pi.
• **Dynamic Geometric Software** – use software/applications to explore angle relationships formed by intersecting lines.
  ○ Geometer's Sketchpad
  ○ GeoGebra

Resources
7.GM.4 - This activity allows you to investigate how the area and circumference of a circle compare to its radius and diameter. http://illuminations.nctm.org/Activity.aspx?id=3547 7. GM. 4 - This is a video that introduces the parts of a circle. http://www.virtualnerd.com/pre-algebra/perimeter-area-volume/circles/define-circles/circle-definition 7.GM.4 - This is an interactive website that helps students develop an understanding of circumference and area of a circle. http://www.pbslearningmedia.org/resource/mapt-math-ee-intcircle/pizza-pi-circumference-and-area-of-a-circle/ 7.GM.4 - Performance Task for calculating area of a circle. https://www.illustrativemathematics.org/content-standards/tasks/34

Sample Formative Assessment Tasks/Questions

7.GM.4, 7.EEI.4b: If the circumference of a circle is 9π, then what would be the area of that circle?

Answer
Students will need to use the circumference of the circle given and the formula of circumference to find the radius of the circle. Once the radius is found, students can use the formula for area of a circle to find the area of this circle.

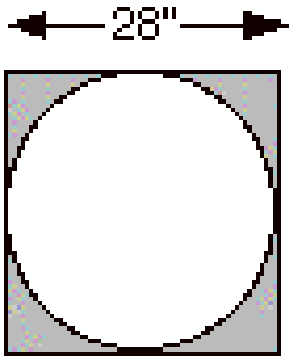
Since $C = 2\pi r$ and the circumference of the circle is 9π, you can substitute 9π for C.

$C = 2\pi r$	
$9\pi = 2\pi r$	Substitute 9π
$9 = 2r$	Divide by π
$4.5 \text{ units} = r$	Divide by 2

Then, substitute the radius in the area formula.

$A = \pi r^2$	
$A = \pi(4.5^2)$	Substitute r
$A = \pi(20.25)$	Divide by π
$A \approx 63.585 \text{ units}^2$	Divide by 2

7.GM.4, 7.EEI.4: If a circle is cut from a square piece of plywood, how much plywood would be left over?

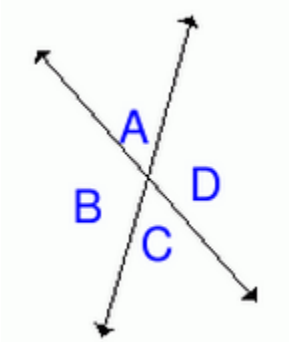


Answer
The area of the square is 28 x 28 or 784 in^2. The diameter of the circle is equal to the length of the side of the square, or 28 inches, so the radius would be 14 inches. The area of the circle would be approximately 615.44 in^2. The difference in the amounts (plywood left over) is approximately 168.56 in^2 (784 –615.44). Once the circle is cut, the area of the plywood is approximately 168.56 in^2.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

7.GM.5, 7.EEI.4b

In the figure on the right, two lines intersect to form $\angle A$, $\angle B$, $\angle C$, and $\angle D$.



If m$\angle A$ = (4x +11)° and m$\angle C$ = 59° , solve for *x* and find the measurements of ∠B and ∠D.

Answer

Since $\angle A$ and $\angle C$, are vertical angles, they have equal measures. Students can write an equation to solve for *x.*

m$\angle A$ = m$\angle C$

4*x* + 11 = 59

x = 12

Since $\angle C$ and $\angle D$ are supplementary angles, their sum is 180 degrees. To find the measure of angle D, subtract 59 from 180.

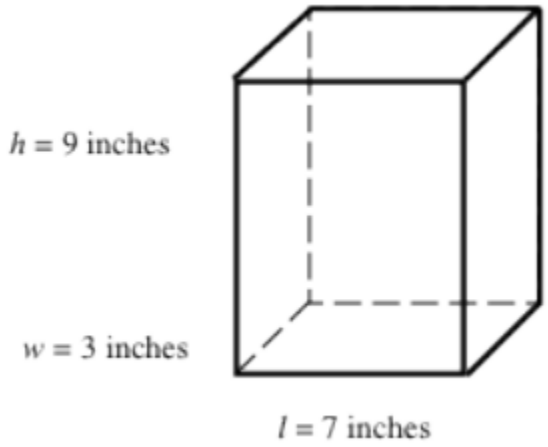
m$\angle C$ + m$\angle D$ = 180

59 + m$\angle D$ = 180

m$\angle D$ = 121°

Since $\angle B$ and $\angle D$, are vertical angles, they have equal measures. Therefore, $\mathrm{m}\angle B = 121°$.

7.GM.6, 7.EEI.4b: Huong covered the box below with sticky-backed decorating paper. The paper costs 3¢ per square inch. How much money will Huong need to spend on paper?



Answer
Students must find the total surface area of the box first. Students can do this by drawing the net of the rectangular solid, find the area of each face, and then finding the total surface area by adding the areas of the faces together. Students can also use a formula for finding the surface area of a rectangular prism.

$SA = 2lw + 2lh + 2wh$
$SA = 2(7 \times 3) + 2(7 \times 9) + 2(3 \times 9)$
$SA = 222 \text{ in}^2$

Next, students must find the total cost of wrapping the package. To do this, multiply the cost, $0.03 per square inch, by the total surface area. Hong will need to spend $6.66 on paper.

Source: NC DPI 7th Grade Mathematics Unpacked Contents

Unit Title
Statistics
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.DSP.1* Investigate concepts of random sampling. a. Understand that a sample is a subset of a population and both possess the same characteristics. b. Differentiate between random and nonrandom sampling. c. Understand that generalizations from a sample are valid only if the sample is representative of the population. d. Understand that random sampling is used to gather a representative sample and supports valid inferences about the population. o Recognize that a random sample is an unbiased representation of a group. o Recognize that a random sample must be used in conjunction with the population to make valid inferences. **7.DSP.2*** Draw inferences about a population by collecting multiple random samples of the same size to investigate variability in estimates of the characteristic of interest. o Important to make sure that the sample sizes of the population are the same. o Issues of variation in the samples should be addressed through investigations of multiple subsets. **7.DSP.3** Visually compare the centers, spreads, and overlap of two displays of data (i.e., dot plots, histograms, box plots) that are graphed on the same scale and draw inferences about this data. o Understand that alternate data sources should be used beyond sports data. o Make inferences about multiple displays of data of the same type of data representation with the same scale (e.g., visually comparing the box plot of company A salaries to the box plot of company B salaries). **7.DSP.4*** Compare the numerical measures of center (mean, median, mode) and variability (range, interquartile range, mean absolute deviation) from two random samples to draw inferences about the populations. o Use the comparisons of data to draw inferences about populations.
New Academic Vocabulary for This Unit
• subset • random sample • non-random sample • Inference

Prior Knowledge Required for this Unit
• Calculating center and spread (6.DS.2) • Creation of dot plots, histograms, and boxplots (6.DS. 4) • Analyze a single data display 6.DS.5
Subsequent Knowledge Related to this Unit
The work students have done from Grade 6 and Grade 7 utilizing univariate data will prepare them for statistics with the use of bivariate data in Grade 8. The plotting of points in Grade 6 will allow the students to work very closely with the most common form of bivariate data, scatter plots. In Grade 8, students will also use their graphing skills to locate the best line of fit for a data set. Working with the best line of fit ties in directly with finding the slope and intercept of a line in Grade 8. In high school math courses, students will begin to identify linear, quadratic, and exponential relationships among data values. Also, in high school math courses, students will determine the regression equations for linear, quadratic, and exponential relationships.
Relationship Among Standards in this Unit
The standards in this unit require students to use previous skills to draw inferences about one population from a random sampling of that population. They will extend these skills to draw informal inferences about two populations.

Potential Instructional Strategies/Lessons

• **Real-world Connections** – Allow students to use real-world examples from social studies and science of random samples to make predictions about population. Additionally, have students represent the data and analyze shape, center, and spread to draw conclusions about population.
• **Technology** - Students can use technology to create data displays.
  ○ Box Plot Tool
  ○ Histogram Tool
  ○ National Library of Virtual Manipulatives

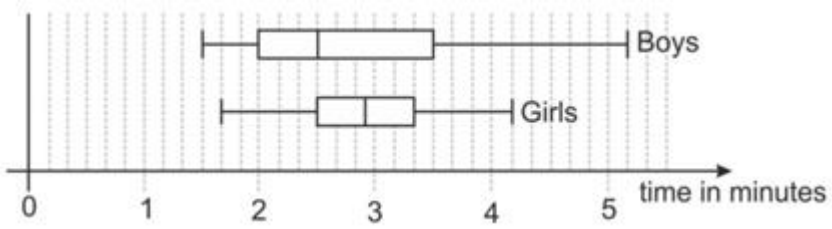
Resources
7.DSP.1, 7.DSP.2 - This activity, from NCTM's Illuminations, engages students as they collect data from classmates and use the data to answer statistical questions regarding their class. http://illuminations.nctm.org/Lesson.aspx?id=4035 7.DSP.3 - This site provides sample problems to use as examples in class that provide dot plots and tables for students to construct box plots and allows them to compare the centers and spreads of two sets of data. http://www.opusmath.com/common-core-standards/7.sp.3-informally-assess-the-degree-of-visual-overlap-of-two-numerical-data
Sample Formative Assessment Tasks/Questions
7.DSP.1: The cafeteria workers at the local high school would like to get more students to eat hot lunch in the cafeteria. The student council has been asked to take on the task of surveying the student body to determine the student's' preferences for hot lunch. The council has determined the complete the survey two different ways. Identify the type of sampling used in each survey option. Which survey option should the student council use and why? a. Write all of the students' names on cards and pull them out in a draw to determine who will complete the survey. b. Survey the first 20 students that enter the lunchroom. *Answer* a. This method of gathering the data is known as random sampling which is the way to gather data from a small population of people to make a decision for a larger group of people. Many random samples should be taken and not just one to make valid inferences about the larger group of data. b. This second method is not the correct way to gather this data because the data will be biased because you are focusing on the first 20 students coming into the cafeteria and not selecting people at random. Source: NC DPI 7th Grade Mathematics Unpacked Contents

7.DSP.2: The table below shows data from two random samples about student choices when it comes to different types of drinks they like to have when they are at the movies. Draw two inferences from the data sets.

Sample	Diet	Regular	None	Total
#1	17	65	18	100
#2	21	60	19	100

Answer
Answers may vary pending on the interpretation of the data. The students surveyed for this data show a liking to regular soda when they are at the movies over diet soda and no soda at all. The number of students that like diet soda is close to the number of students that do not like soda at all when at the movies.

7.DSP.3: The box-and-whisker plots below represent the times taken by a school class to complete an obstacle course. The times have been separated into boys and girls. The boys and the girls each think that they did the best. Based upon the box-and-whisker plot give reasons on why the boys and girls would say they did the best.



Answer
BOYS: The boys did have the fastest time at 1 minute and 30 seconds, so the fastest individual time was a boy. The boys also had the smaller median, 2 minutes and 30 seconds meaning half of the boys were finished when only one fourth of the girls were finished (since the girls' first quartile is also 2:30). In other words, the boys' average time was faster.
GIRLS: The boys had the slowest time at 5 minutes and 10 seconds so by the time all the girls were finished there was still at least one boy completing the course. The girls had the smaller third quartile at 3 minutes and 20 seconds meaning that even without taking the slowest fourth of each group into account, the girls were still quickest.

7.DSP.4: The two data sets below depict random samples of the management yearly salaries in two companies. Based on the salaries below which measure of center will provide the most accurate estimation of the salaries for each company?
Company A: $1.2 million; $242,000; $265,500; $140,000; $281,000; $265,000; $211,000
Company B: $5 million; $154,000; $250,000; $250,000; $200,000; $160,000; $190,000

Answer
The median would be the most accurate measure since both companies have one value in the million that is far from the other values and would affect the mean.

Unit Title
Probability
Content Standards with Clarifying Notes *Open bullets indicate clarifying notes.*
7.DSP.5 Investigate the concept of probability of chance events. a. Determine probabilities of simple events. b. Understand that probability measures likelihood of a chance event occurring. c. Understand that the probability of a chance event is a number between 0 and 1. d. Understand that a probability closer to 1 indicates a likely chance event. e. Understand that a probability close to $\frac{1}{2}$ indicates that a chance event is neither likely nor unlikely. f. Understand that a probability closer to 0 indicates an unlikely chance event. ○ Write the probability of an event as the ratio of possible outcomes to the total number of outcomes ($\frac{possible\ outcomes}{total\ number\ of\ outcomes}$). **7.DSP.6*** Investigate the relationship between theoretical and experimental probabilities for simple events. a. Determine approximate outcomes using theoretical probability. b. Perform experiments that model theoretical probability. c. Compare theoretical and experimental probabilities. ○ Recognize the relationship between experimental and theoretical probability. Students need to see that experimental probability proves theoretical probability. ○ Use experimental probability $\left(\frac{number\ of\ times\ an\ event\ occurs}{total\ number\ of\ trials}\right)$to perform experiments that can be compared to theoretical probability $\left(\frac{number\ of\ ways\ an\ event\ can\ occur}{total\ number\ of\ outcomes}\right)$. ○ Find probabilities for simple events using methods such as organized lists and tables.

7.DSP.7* Apply the concepts of theoretical and experimental probabilities for simple events.
a. Differentiate between uniform and non-uniform probability models (distributions).
b. Develop both uniform and non-uniform probability models.
c. Perform experiments to test the validity of probability models.
- Understand that a uniform probability model is an example of a model where all events are equally likely (e.g., a standard deck of 52 cards has four suits, each suit is equally likely to be drawn from the deck of cards).
- Understand that a non-uniform probability model is an example of a model where all events are not equally likely (e.g., a spinner that does not have equal distribution or sector sizes).

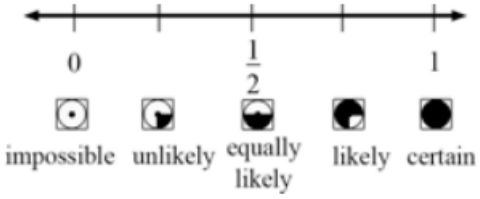
7.DSP.8* Extend the concepts of simple events to investigate compound events.
a. Understand that the probability of a compound event is between 0 and 1.
b. Identify the outcomes in a sample space using organized lists, tables, and tree diagrams.
c. Determine probabilities of compound events using organized lists, tables, and tree diagrams.
d. Design and use simulations to collect data and determine probabilities.
e. Compare theoretical and experimental probabilities for compound events.
- Understand that compound events represent two or more events happening at the once.
- Find the probability of a compound event by adding the probability of the events ($P(A, B) = P(A) + P(B)$).
- Explain that the fraction of outcomes in probability of a compound event is similar to the probability of a simple event.
- Represent probability outcomes as fractions, decimals, or percents.

New Academic Vocabulary for This Unit
• Probability • Theoretical Probability • Experimental Probability • Chance • Likelihood • Uniform, non-uniform probability models • Simple events • Compound events • certain • impossible • Sample Space • Simulation • Equally Likely • Equivalent

Prior Knowledge Required for this Unit
• Write a ratio (6.RP.1) • Multiplication of fractions (5.NSF.5, 5.NSF.6) • Addition of fractions (5.NSF.1)
Subsequent Knowledge Related to this Unit
In high school math courses, students will apply their knowledge of probability to determine the likelihood of an object being located in an indicated area of a geometric figure. Students will also extend their understanding of probability to include conditional probability.
Relationship Among Standards in this Unit
Standards in this unit will establish an understanding of relationships that exist among theoretical, experimental, simple, and compound probabilities.

Potential Instructional Strategies/Lessons

- **Real-world Connections**
  - Conduct an experiment using a Styrofoam cup by tossing the cup and recording how it lands.
    - How many trials were conducted?
    - How many times did the cup land right side up?
    - How many times did the cup land upside down?
    - How many times did the cup land on its side?
    - Determine the probability for each result.
  - Conduct an experiment using a coin and a spinner and record results in a table. (Compound Event)*(Students could determine the probability of rolling a certain number, let's say 2, and a certain color (if using a color wheel) or the same number on a number wheel)*
    - Have students conduct at least 20 trials
    - How many times did you land on the number 2 and the color red?
    - How many times did you not land on the number 2 and not get the color red?
    - Determine the probability for the compound event with both desired outcome and with non-desired outcomes. Together the sum of those probabilities should equal 1.
  - Give students a copy of a menu from a restaurant (great small group assignment). Have the groups list out the number of choices for drinks, an appetizer, and an entree. You could also include side choices, if desired. Have students list all possible ordering combinations using a tree diagram, if someone ordered a meal that includes one drink, one appetizer, and one entree. (Source: NC DPI 7th Grade Mathematics Unpacked Contents)
- **Number Line** - Students recognize that the probability of any single event can be can be expressed in terms such as impossible, unlikely, likely, or certain or as a number between 0 and 1.



(Source: NC DPI 7th Grade Mathematics Unpacked Contents)

- **Technology** – Technology can be used to simulate the act of using coins, spinners, and fair number cubes to determine experimental probability.
  - Marble Mania
  - NCTM's Random Drawing Tool

Resources
7.DSP.5 - This site provides a SMART notebook lesson to introduce the idea of probability. http://exchange.smarttech.com/details?id=e774ee87-e207-4167-9347-4bbddfabc454 7.DSP.5, 7.DSP.6, 7.DSP.7, 7.DSP.8 - This site provides a lesson plan for activity stations allowing students to express their understanding of theoretical, experimental, simple, and compound probabilities. https://ilearn.marist.edu/access/content/user/10026480@marist.edu/edTPA/Lesson%20Plan%20_4.pdf 7.DSP.5, 7.DSP.6, 7.DSP.7, 7.DSP.8 - This site provides several lesson plan and activities that will allow students an opportunity to express their understanding of theoretical, experimental, simple, and compound probabilities. https://www.engageny.org/sites/default/files/resource/attachments/math-g7-m5-teacher-materials.pdf 7.DSP.5, 7.DSP.6, 7.DSP.7 - This site allows students to watch and listen to an explanation of different ways to express probability set to hip-hop music. This video focuses on the basics of odds and probability and expressing probabilities as decimals, percents, and fractions. http://www.pbslearningmedia.org/resource/mgbh.math.sp.future/probability-tell-the-future/
Sample Formative Assessment Tasks/Questions
7.DSP.5-8: Evaluating Statements About Probability - The task will help you assess how well students understand concepts of: equally likely events, randomness, and sample size. (Source: Mathematics Assessment Resource Service)

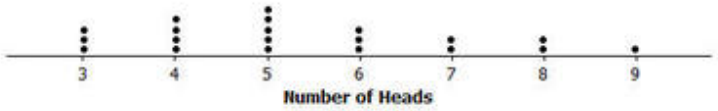
7.DSP.6: Each of the 20 students in Mr. Anderson's class flipped a coin ten times and recorded how many times it came out heads.

a. How many heads do you think you will see out of ten tosses?
b. Would it surprise you to see 4 heads out of ten tosses? Explain why or why not.
c. Here are the results for the twenty students in Mr. Anderson's class. Use this data to estimate the probability of observing 4, 5 or 6 heads in ten tosses of the coin. (It might help to organize the data in a table or in a dot plot first.)

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number of heads	3	5	4	6	4	8	5	4	9	5	3	4	7	5	8	6	3	6	5	7

Answer

a. Most students will reply that they expect to see 5 heads, but encourage answers like "around 5 heads".
b. It would not be surprising to see only 4 heads in ten tosses. Although this outcome might occur less often than getting 5 heads, students should understand that there will be variability in the outocmes when a coin is tossed ten times, and that you don't always get 5 heads.
c. It helps to organize the data before estimating the requested probability. A dot plot of the data is shown here:



Because 12 of the observed outcomes were 4, 5 or 6, the estimated probability is 12/20 = 0.60.

Source: Illustrative Mathematics

7.DSP.7: Is it Fair? - In this task, students will play the game in groups and record their information. Students will then use probability to determine whether they feel the game is fair or not.

Source: Georgia Department of Education

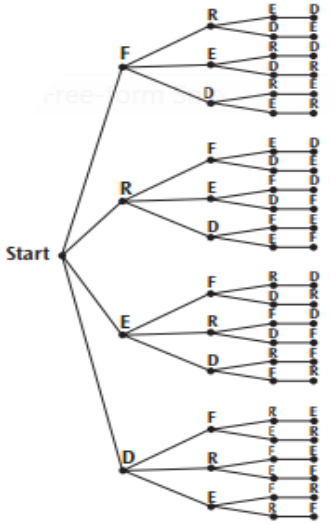
7.DSP.8: How many ways could the students, Amy, Ben, and Lorenzo, come in 1st, 2nd, and 3rd place?

Answer
Making an organized list will identify that there are 6 ways for the students to win a race.
A, B, L
A, L, B
B, L, A
B, A, L
L, A, B
L, B, A

Source: NC DPI 7th Grade Mathematics Unpacked Contents

7.DSP.8: Using a tree diagram, show all possible arrangements of the letters in the name FRED. If each of the letters is on a tile and drawn at random, what is the probability that you will draw the letters F-R-E-D in that order? What is the probability that your "word" will have an F as the first letter?

Answer
A tree diagram reveals that, out of 24 total outcomes, there is only one outcome where the letters F-R-E-D appear in that order, so the probability of the event occurring is $\frac{1}{24}$. Regarding the second question, the entire top branch (6 outcomes) represents the outcomes where the first letter is F, so the probability of that occurring is $\frac{6}{24} = \frac{1}{4}$.



Source: California Mathematics Framework for Grade 7